ECE 344
Microwave Fundamentals

Lecture 05 – Part2:
Impedance Matching

Prepared By

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Single-stub tuning

We consider a matching technique that uses a single open-circuited or short-circuited length of transmission line (a "stub"), connected either in parallel or in series with the transmission feed line at a certain distance from the load. Such a tuning circuit is convenient from a microwave fabrication aspect, since lumped elements are not required. The shunt tuning stub is especially easy to fabricate in microstrip or stripline form.
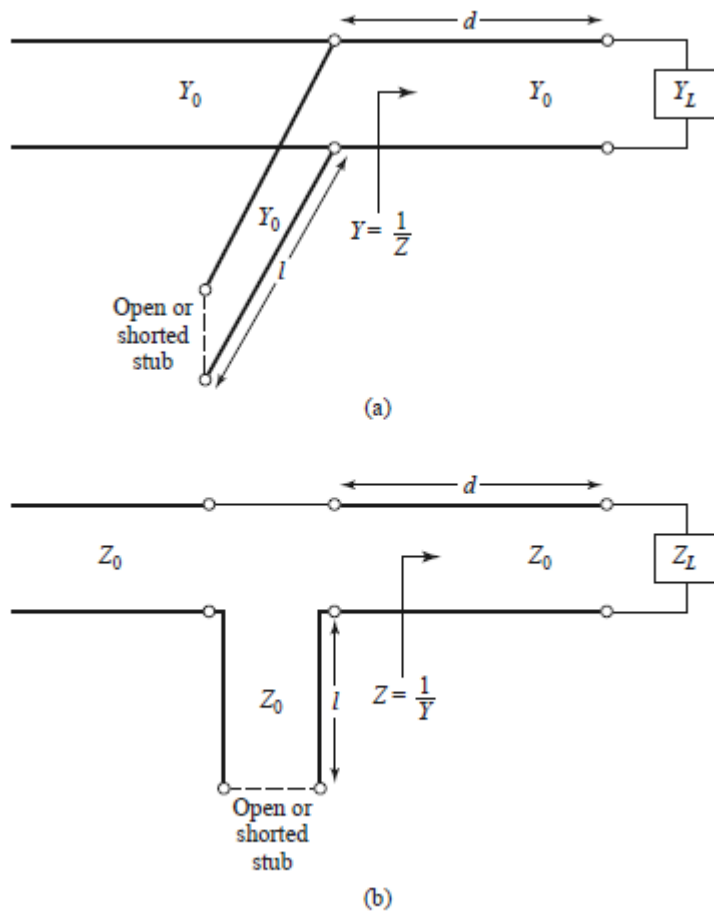

Figure: Single-stub tuning circuits.
(a) Shunt stub. (b) Series stub.

Single-stub tuning circuits. (a) Shunt stub. (b) Series stub.

In single-stub tuning, the two adjustable parameters are the distance, d, from the load to stub position, and the value of susceptance or reactance provided by the shunt or series stub.

For the shunt-stub case, the basic idea is to select d so that the admittance, Y, seen looking into the line at distance d from the load is of the form **Y_0+jB.** Then the stub susceptance is chosen as –jB, resulting in a matched condition.
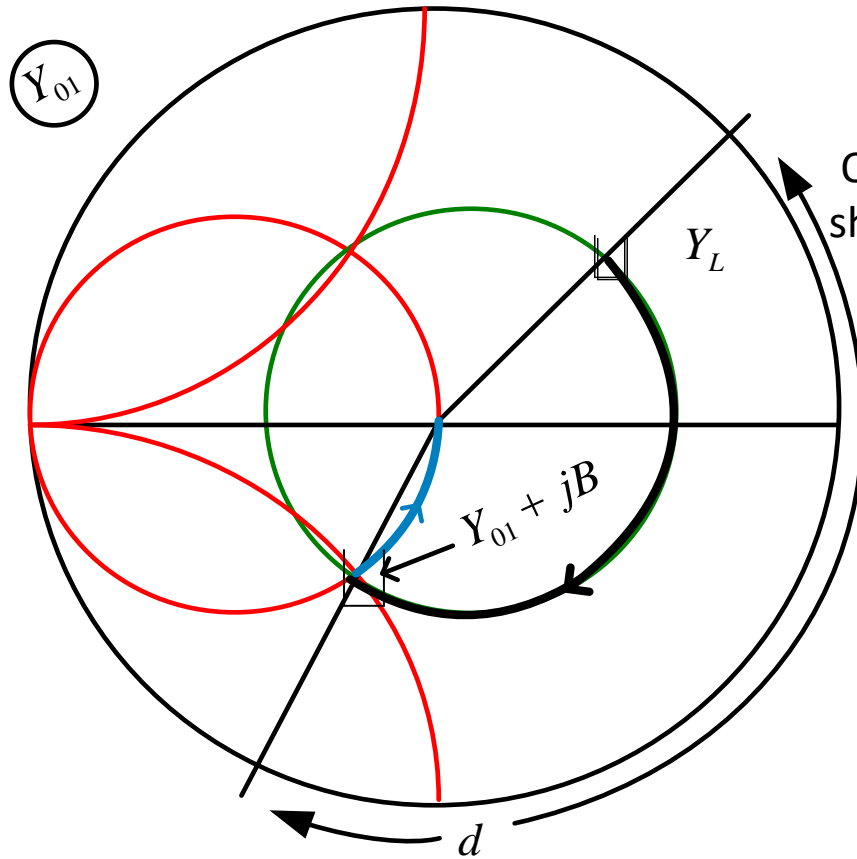
For the series stub case, the distance d is selected so that the impedance, Z, seen looking into the line at a distance d from the load is of the form **Z_0+jX**. Then the stub reactance is chosen as –jX, resulting in a matched condition.
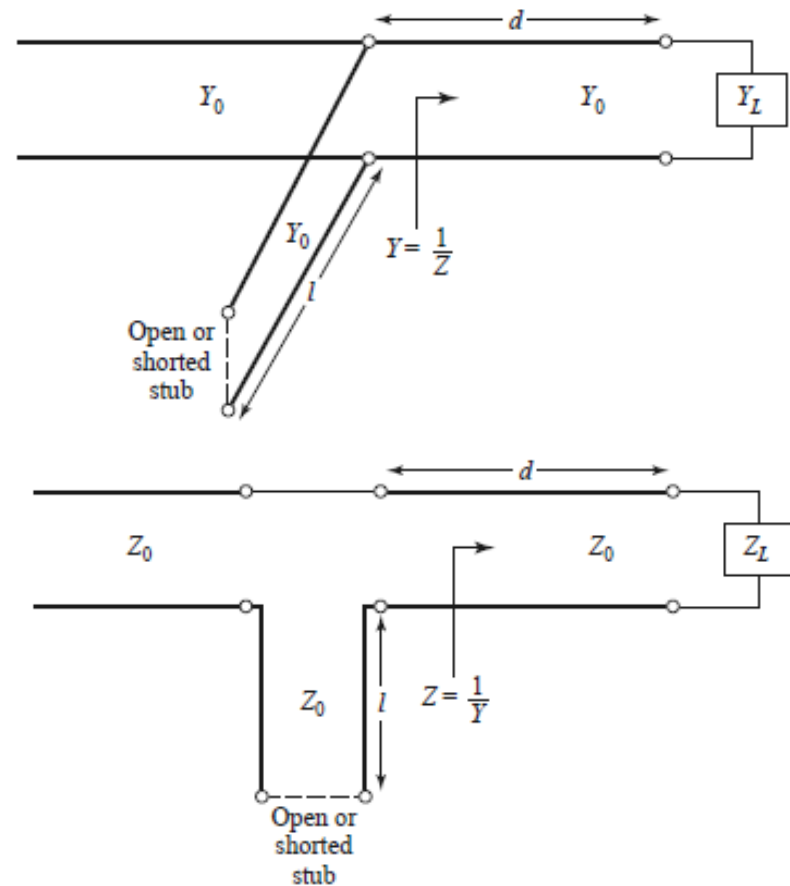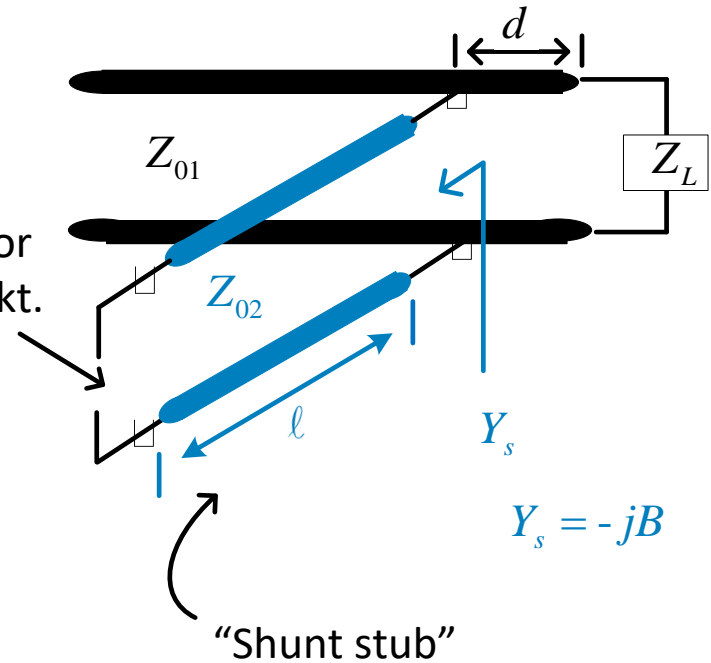
We can determine the two adjustable parameters using Smith chart and analytic solutions. The Smith chart solutions are fast, intuitive, and usually accurate enough in practice. The analytic expressions are more accurate, and useful for computer analysis.
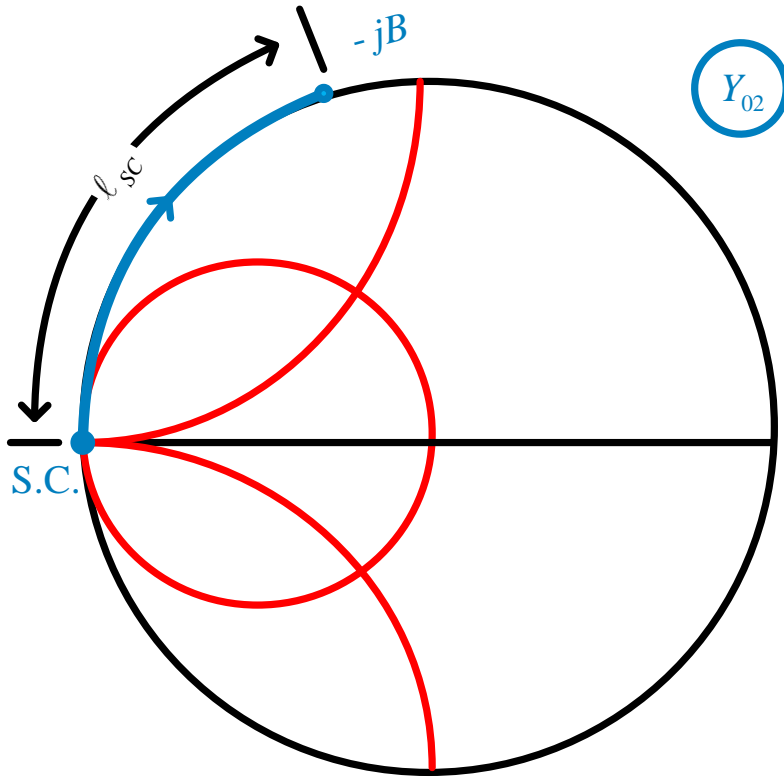
Impedance matching technique

Transmission-Line Matching

Single-Stub Tuning

Y_{01}

Y_L

$Y_{01} + jB$

Open or short ckt.

d

Γ plane

Z_{01}

Z_L

Z_{02}

ℓ

Y_s

$Y_s = -jB$

"Shunt stub"

Note: Only one of two possible solutions is shown.

4

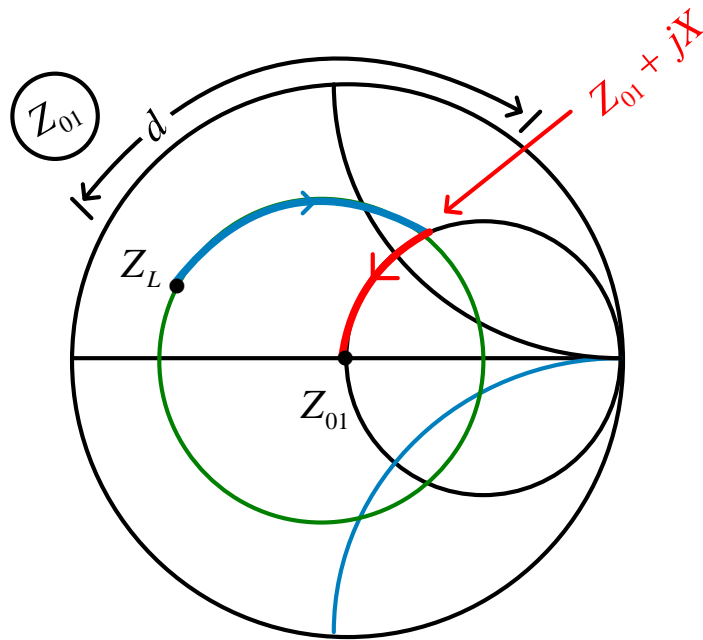# Transmission-Line Matching (cont.)

Short ckt. Load on stub

Y_{02}

$-jB$

ℓ_{SC}

S.C.

Γ plane

Open ckt. Load on stub

$-jB$

O.C.

ℓ_{OC}
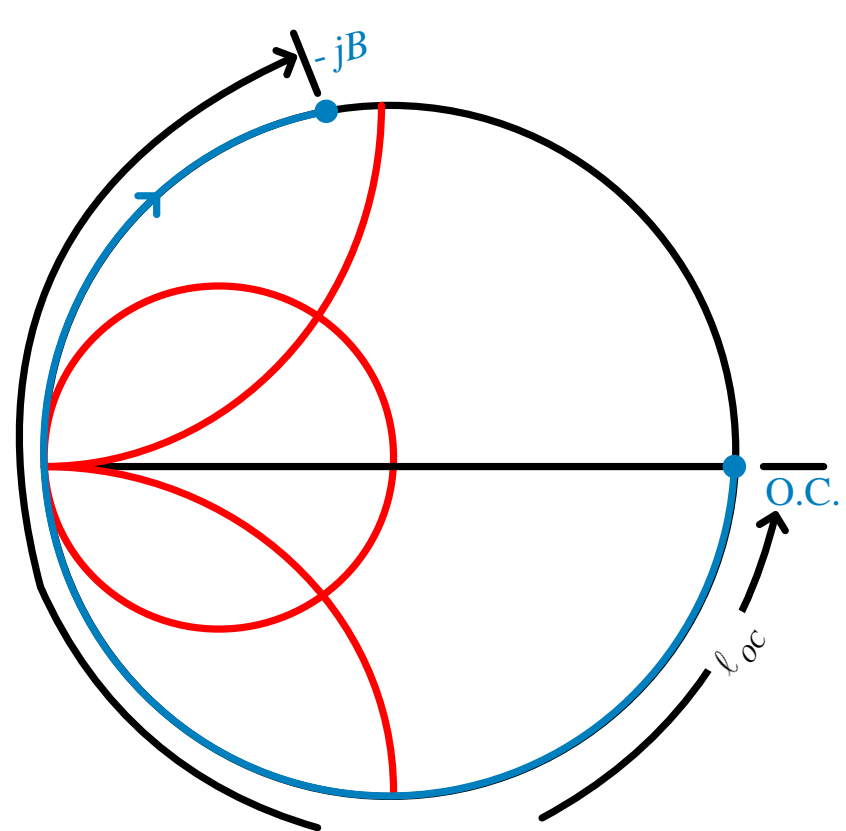
Γ plane

Transmission-Line Matching (cont.)

Series stub

Γ plane

S.C.

ℓ_{SC}

$-jX$

$Z_{01} + jX$

d

Z_{01}

Z_{01}

Z_L

Z_{02}

$-jX$

ℓ

Short ckt.

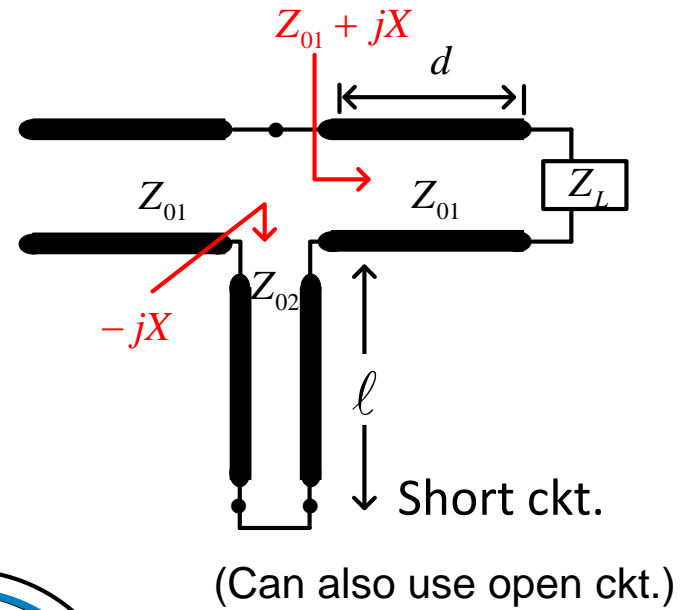(Can also use open ckt.)
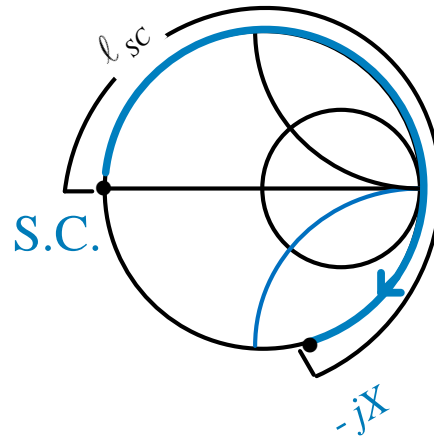
Note: Only one of two possible
solutions is shown.

Example: Single-stub shunt tuning

For a load impedance Z_L=15+j10 Ω, design two single-stub shunt tuning networks to match this load to a 50 Ω line. Assuming that load is matched at 2 GHz, and the load consists of a resistor and inductor in series, plot the reflection coefficient magnitude from 1 GHz to 3 GHz for each solution.
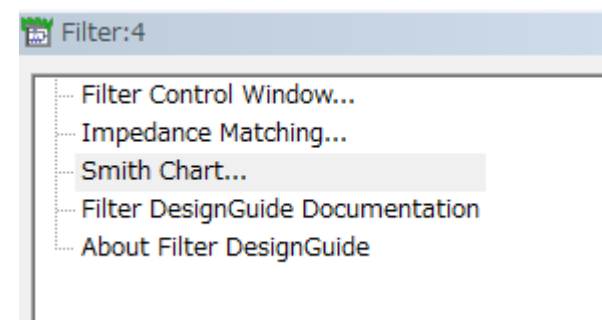
Solution:

Here, we use ADS tool to get the two networks.

DesignGuide → Filter →

Filter:4
- Filter Control Window...
- Impedance Matching...
- Smith Chart...
- Filter DesignGuide Documentation
- About Filter DesignGuide

Switch for admittance and impedance Smith Charts

Impedance matching technique

First input frequency=2 GHz
Select Z_L and input Z_L=15+j10
Select transmission component
Rotate clockwise to make Y=1-j1.33
You can get the rotation angle is 15.809 Deg

$$d_1 = 15.809 / 360 * \lambda = 0.044\lambda$$

Impedance matching technique

9

Open stub

$$Z_L = \infty$$

Maximum value: 9950
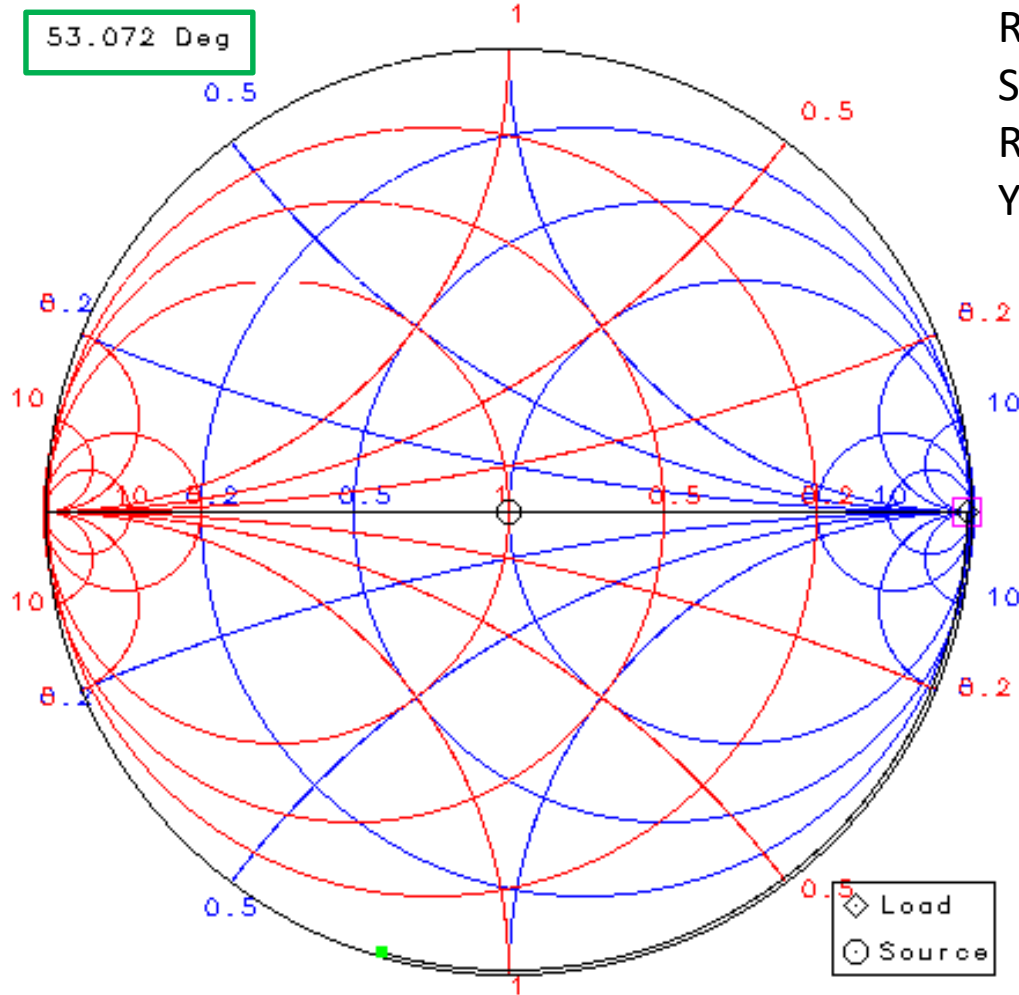
Reset

Select transmission component

Rotate clockwise to make Y=j1.33

You can get the rotation angle is 53.072 Deg
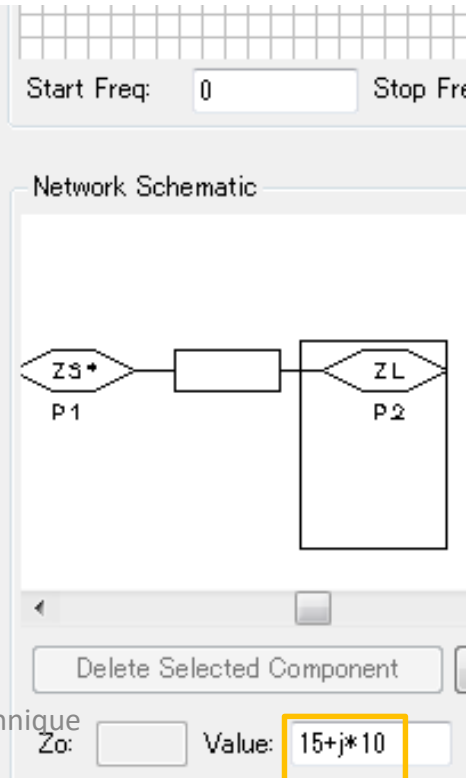
$$l_1 = 53.091/360 * \lambda = 0.147\lambda$$

Freq (GHz)

`2`

Z0 (Ohms)

`50`

☑ Normalize

Current Schematic

[smith_chart_prj]: 1

SmartCo

DA_Smit

Define Source/Load Network Terminations...

139.572 Deg

Reset
Select Z_L and input Z_L=15+j10
Select transmission component
Rotate clockwise to make Y=1+j1.33
You can get the rotation angle is 139.572 Deg

$$d_1 = 139.572/360 * \lambda = 0.388\lambda$$

Start Freq: `0`   Stop Fr

Network Schematic

◇ Load
⊙ Source

☐ Lock Source Impedance   ☐ Lock Load Impedance

Gamma: `0.55350` < `-123.835`   **Z:** `0.36076` +j `-0.47823`

VSWR: `3.47925`   **Y:** `1.00531` +j `1.33267`   Impedance matching technique   **Zoi**   **Value:** `15+j*10`

Delete Selected Component

Palette

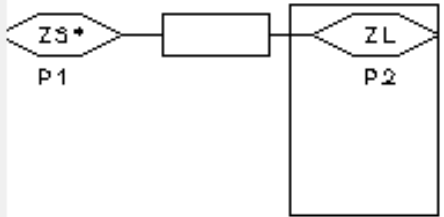Freq (GHz): `2` Z0 (Ohms): `50` ☑ Normalize

Current Schematic: `[ test_a_prj ]: 1`
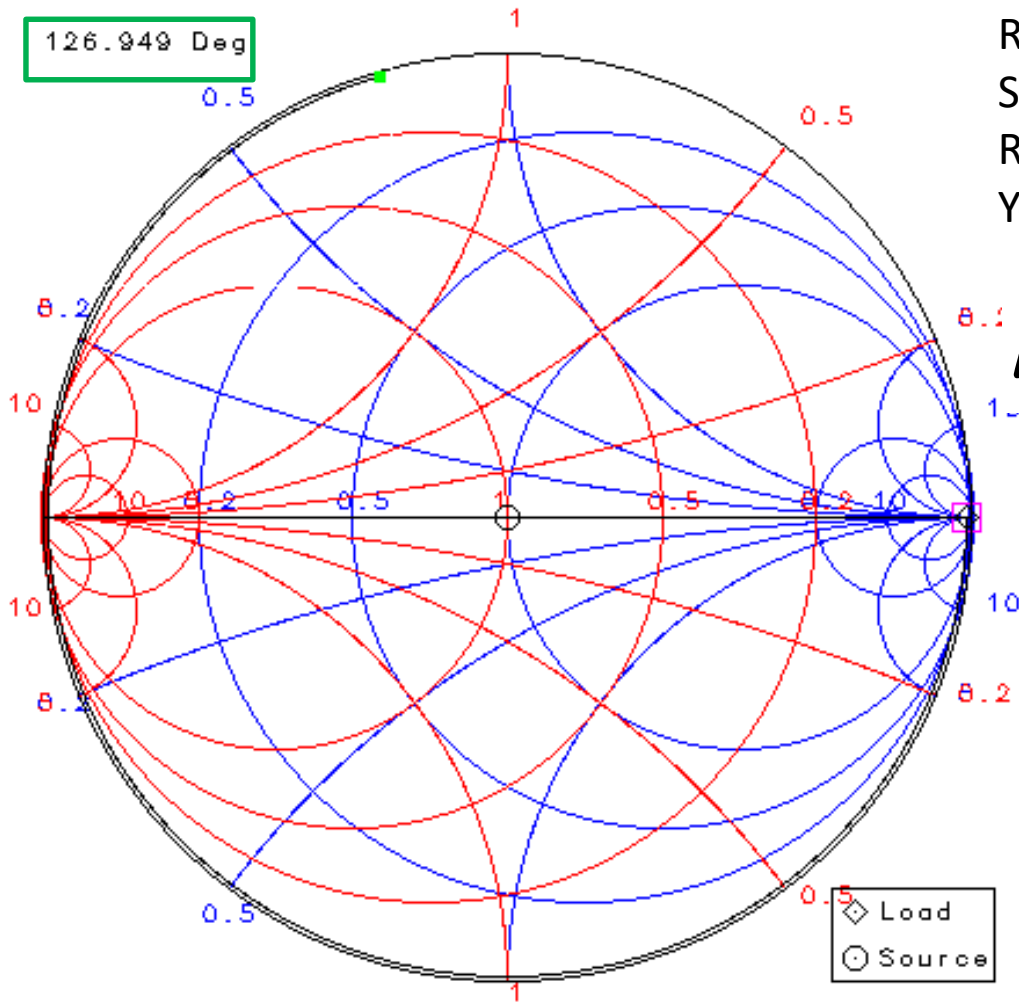
SmartCompo...

Define Source/Load Network Terminations...

126.949 Deg

Open stub

$$Z_L = \infty$$

Reset
Select transmission component
Rotate clockwise to make Y=-j1.33
You can get the rotation angle is 126.949 Deg

$$l_2 = 126.949 / 360 * \lambda = 0.353\lambda$$

Start Freq: `0` Stop Freq:

Network Schematic

ZS* ─── ZL
P1   P2
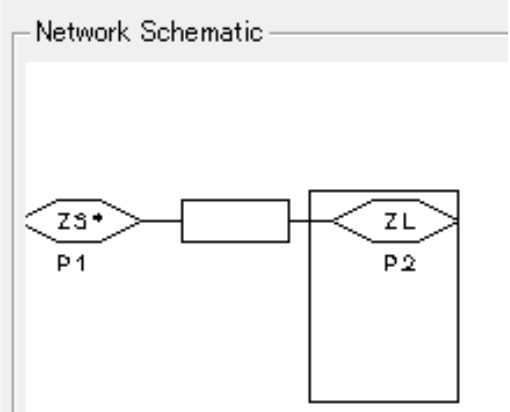
◇ Load
⊙ Source

Palette

☐ Lock Source Impedance   ☐ Lock Load Impedance

Gamma: `0.99000` < `106.101`   Z: `0.00787` +j `0.75214`

VSWR: `199.000`   Y: `0.01391` +j `-1.32940` Zo: Value: `9950+j*0`

Delete Selected Component

Impedance matching technique
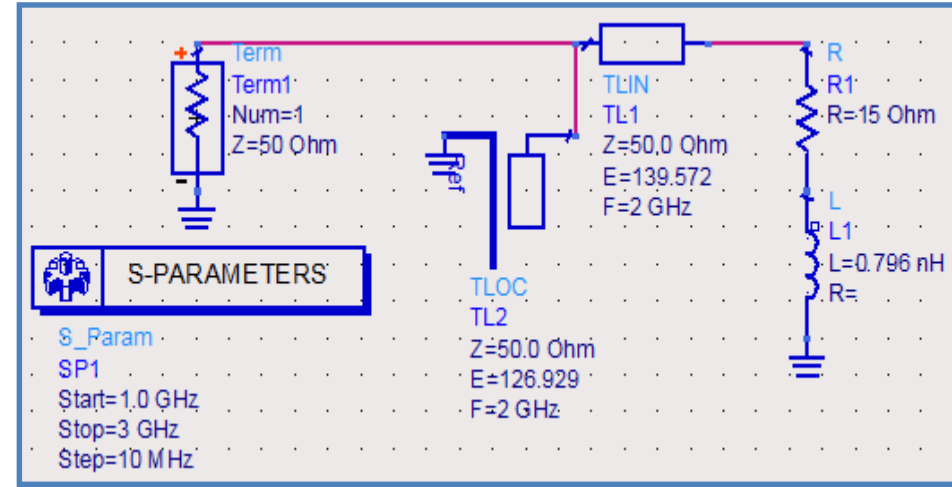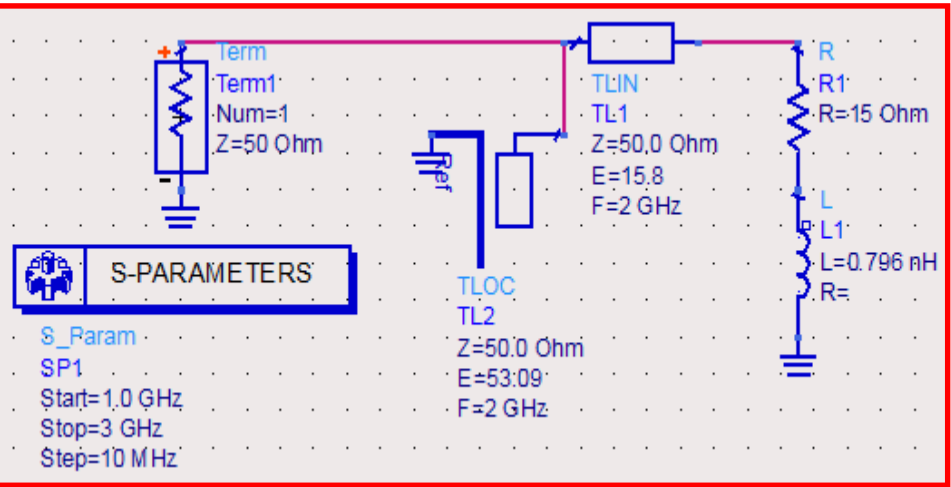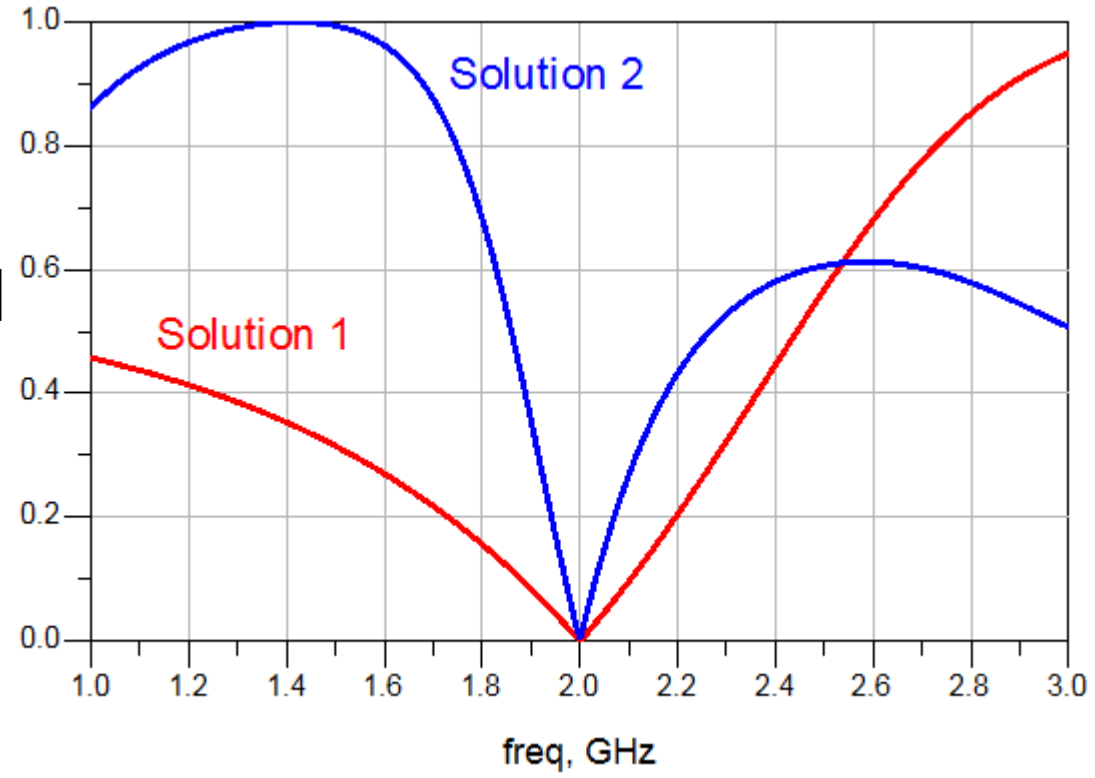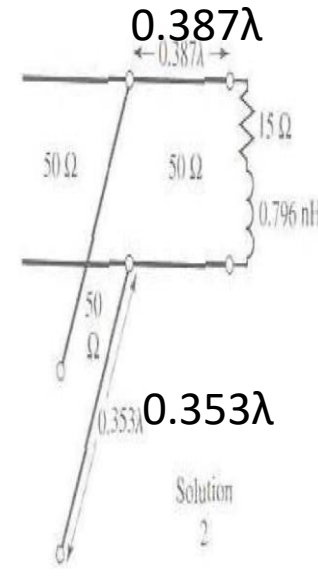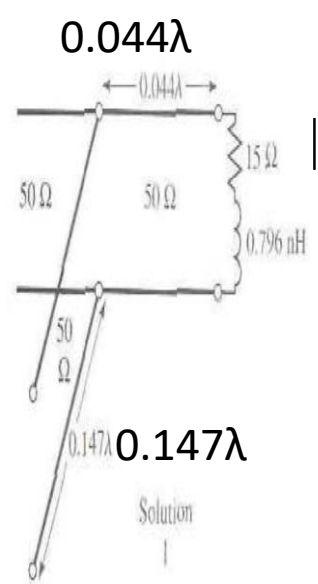
Figure: Reflection coefficient magnitude versus frequency for the tuning circuits.

Impedance matching technique

13

Analytic Solution

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$$G_{in} = Y_0 \frac{Y_L + jY_0 \tan \beta l}{Y_0 + jY_L \tan \beta l}$$

Open or shorted stub
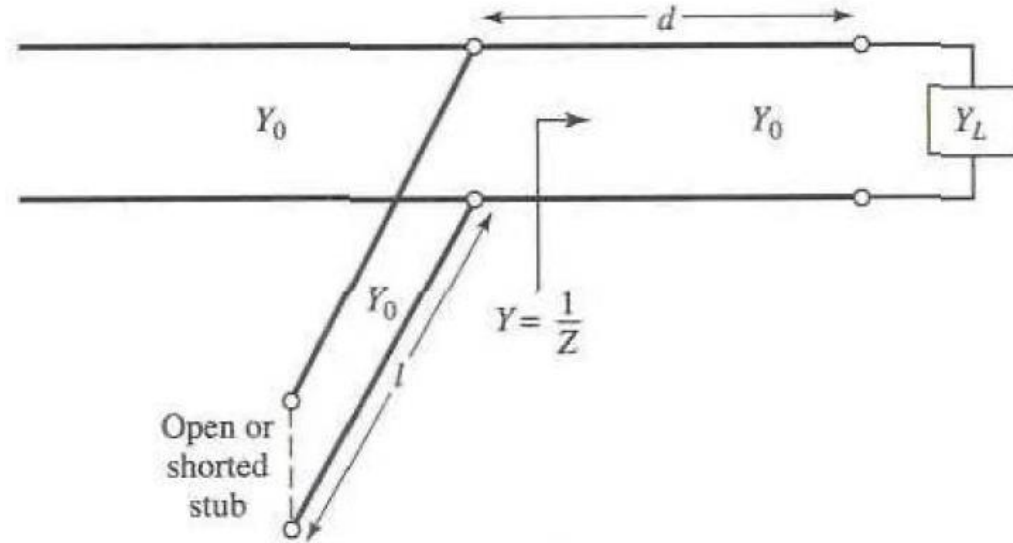
$$Y = G_0 \frac{G_L + jB_L + jG_0 t}{G_0 + j(G_L + jB_L)t} = G_0 \frac{G_L + j(B_L + G_0 t)}{G_0 - B_L t + jG_L t} = G_0 \frac{\left[G_L + j(B_L + G_0 t)\right]\left[G_0 - B_L t - jG_L t\right]}{(G_0 - B_L t)^2 + (G_L t)^2}$$

$$= G_0 \frac{G_L(G_0 - B_L t) - jtG_L^2 + j(B_L + G_0 t)(G_0 - B_L t) + (B_L + G_0 t)G_L t}{(G_0 - B_L t)^2 + (G_L t)^2}$$

$$= G_0 \frac{G_L G_0 (1 + t^2) + j\left(B_L G_0 - B_L^2 t + G_0^2 t - B_L G_0 t^2 - tG_L^2\right)}{(G_0 - B_L t)^2 + (G_L t)^2}$$

where $t = \tan \beta d$

Impedance matching technique

14

In order to get a matched condition, we will choose d (which implies t) to make the real part of the admittance Y to be equal to Y_0, then we have

$$G_L G_0 (1+t^2) = \left(G_0 - B_L t\right)^2 + \left(G_L t\right)^2$$

solving for t gives

$$t = \frac{G_0 B_L \pm \sqrt{G_0^2 B_L^2 - (G_L^2 + B_L^2 - G_L G_0)\left(G_0^2 - G_L G_0\right)}}{(G_L^2 + B_L^2 - G_L G_0)}$$

$$= \frac{G_0 B_L \pm \sqrt{G_L G_0 \left[(G_L - G_0)^2 + B_L^2\right]}}{(G_L^2 + B_L^2 - G_L G_0)}$$

Thus, the two principal solutions for d are

$$\boxed{t = \tan \beta d}$$

$$\frac{d}{\lambda} = \frac{1}{2\pi} \begin{cases} \tan^{-1} t & \text{for } t \geq 0 \\ \pi + \tan^{-1} t & \text{for } t < 0 \end{cases}$$

To find the required stub lengths, first use t in the above equation to find the stub susceptance, B_s=-B, where B is defined as follows:

$$B = G_0 \frac{\left(B_L G_0 - B_L^2 t + G_0^2 t - B_L G_0 t^2 - t G_L^2\right)}{\left(G_0 - B_L t\right)^2 + \left(G_L t\right)^2} = G_0 \frac{B_L G_0 (1-t^2) + t(G_0^2 - G_L^2 - B_L^2)}{\left(G_0 - B_L t\right)^2 + \left(G_L t\right)^2}$$

Impedance matching technique

15

The open-circuited stub length is found as

$$\frac{l_0}{\lambda} = \frac{1}{2\pi} \tan^{-1}\left(\frac{B_s}{Y_0}\right) = \frac{-1}{2\pi} \tan^{-1}\left(\frac{B}{Y_0}\right)$$

$$Z_{ino} = -jZ_0 \cot \beta l$$

$$jB_s = \frac{1}{Z_{ino}} = \frac{1}{-jZ_0 \cot \beta l} = jY_0 \tan \beta l$$

While the short-circuited stub length is found as

$$\frac{l_s}{\lambda} = \frac{-1}{2\pi} \tan^{-1}\left(\frac{Y_0}{B_s}\right) = \frac{1}{2\pi} \tan^{-1}\left(\frac{Y_0}{B}\right)$$

If the length given by above formulas is negative, λ/2 can be added to give a positive result.

$$Z_{ins} = jZ_0 \tan \beta l$$

$$jB_s = \frac{1}{Z_{ins}} = \frac{1}{jZ_0 \tan \beta l}$$

$$= -jY_0 \frac{1}{\tan \beta l} \Rightarrow \tan \beta l = -\frac{Y_0}{B_s}$$

Using the above equations, we can calculate the above example.

$$t_1 = 0.28472$$
$$t_2 = -0.85694$$

➡

$$\frac{d_1}{\lambda} = 0.044$$
$$\frac{d_2}{\lambda} = 0.387$$

➡

$$B_1 = -1.33G_0$$
$$B_2 = 1.33G_0$$

➡

$$\frac{l_1}{\lambda} = 0.147$$
$$\frac{l_2}{\lambda} = 0.353$$

We can see that the same results are obtained.

Homework-1

Design three different matching networks (single L-C matching circuits, single series stub, and single shunt stub) from 3-Ω to 50-Ω, where the center frequency is 2.4 GHz. Plot the frequency responds using ADS tool and compare their bandwidths.
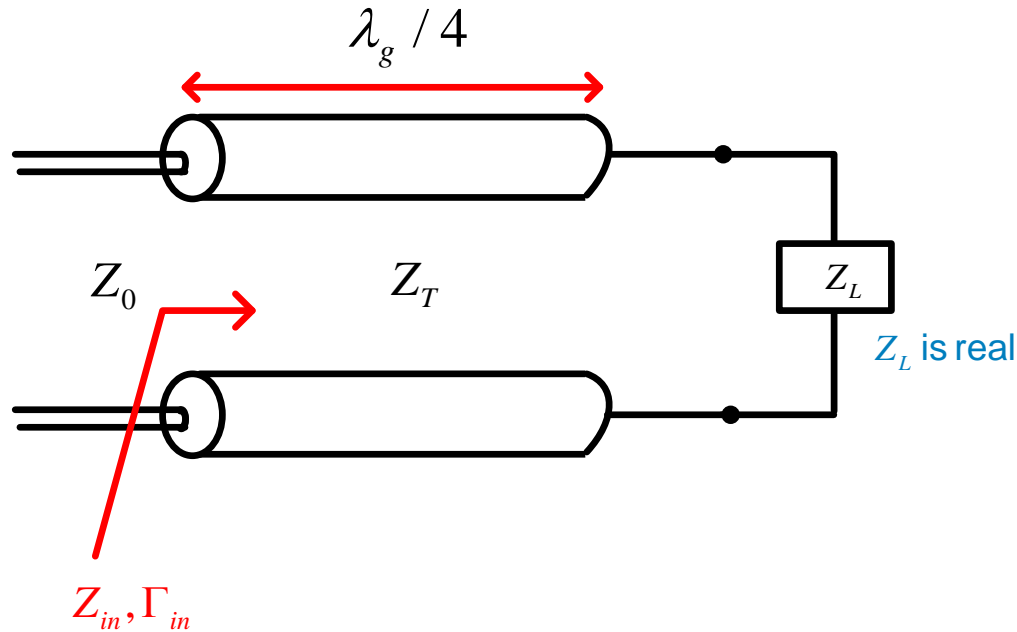

Matching section
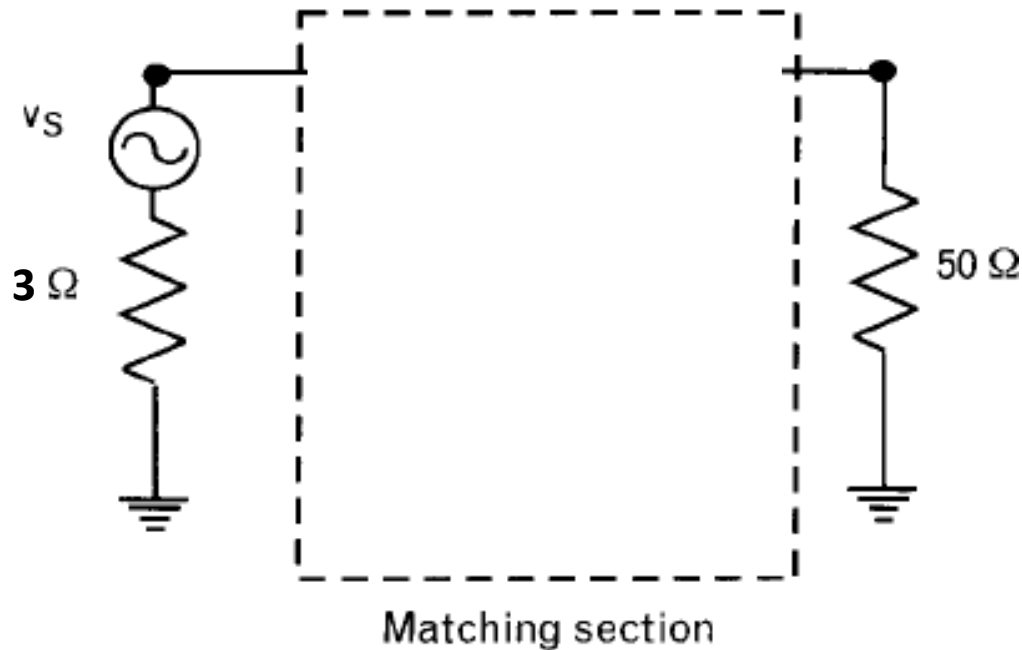
Impedance matching technique

Quarter-Wave Transformer

$\dfrac{\lambda_g}{4}$

Z_0

Z_T

Z_L

Z_L is real

Z_{in}, Γ_{in}

$@ \ f = f_0$

$$\beta\ell = \frac{2\pi}{\lambda_g}\frac{\lambda_g}{4} = \frac{\pi}{2}$$

$$\Gamma_{in} = 0 \quad \text{when} \quad Z_T = \sqrt{Z_0 Z_L}$$

$$Z_{in} = \frac{Z_T^2}{Z_L}$$

Only true at f_0 where $\ell = \lambda_g / 4$

Note: If Z_L is not real, we can always add a reactive load in series or parallel to make it real (or add a length of transmission line between the load and the transformer to get a real impedance).

Quarter-Wave Transformer

A quarter wave transformer, like low-frequency transformers, changes the impedance of the load to another value so that matching is possible.
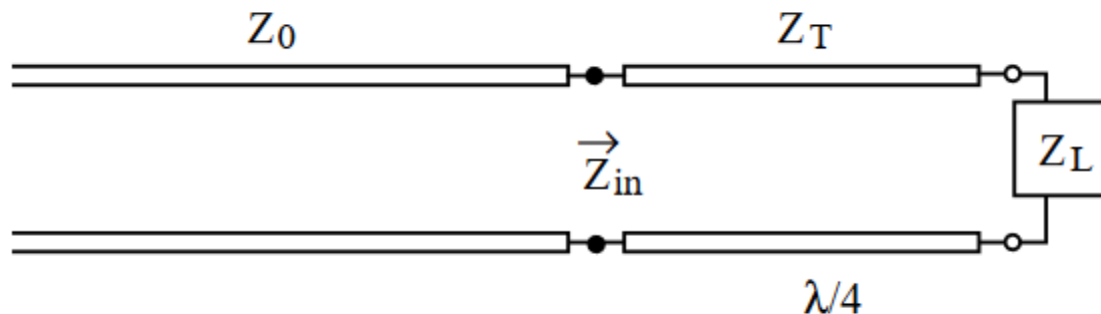
A quarter-wave transformer uses a section of line of characterstic impedance Z_T of $\frac{\lambda}{4}$ long. To have a matching condition, we want $Z_{in} = Z_0$.
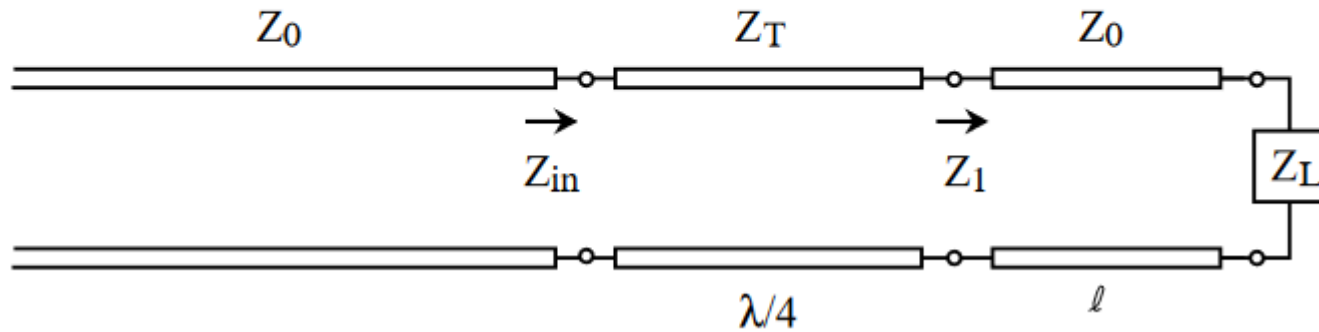
$$Z_{in} = Z_T \frac{Z_L + jZ_T \tan \frac{\pi}{2}}{Z_T + jZ_L \tan \frac{\pi}{2}} = \frac{Z_T^2}{Z_L}, \tag{1}$$

since $\tan \beta l = \tan \frac{2\pi}{\lambda} \frac{\lambda}{4} = \tan \frac{\pi}{2} = \infty$. In order for $Z_{in} = Z_0$, we need that

$$Z_T^2 = Z_0 Z_L \Rightarrow Z_T = \sqrt{Z_0 Z_l}. \tag{2}$$

Quarter-Wave Transformer

If Z_0 and Z_L are both real, then Z_T is real, and we can use a lossless line to perform the matching. If Z_L is complex, it can be made real by adding a section of line to it.

Quarter-Wave Transformer

Example

Given that $Z_L = (30 + j40)\Omega$, $Z_0 = 50\Omega$, find the shortest l and Z_T so that the above circuit is matched. Assume that Z_T is real and lossless.

We want Z_1 to be real and Z_{in} to be $Z_0 = 50\Omega$ in order for Z_T to be real and the matching condition satisfied. We find that $Z_{nL} = 0.6 + j0.8$. In order to make Z_{n1} real, the shortest l from the Smith Chart is $\frac{\lambda}{8}$. Then $Z_{n1} = 3.0$, and $Z_1 = 150\Omega$. Since $Z_{in} = 50\Omega$, we need

$$Z_T = \sqrt{Z_{in}Z_1} = \sqrt{50 \times 150} = 86.6\Omega$$

in order for matching condition to be satisfied.

Quarter-Wave Transformer